Preliminary Considerations for Rational Argumentation under Aleatory and Epistemic Uncertainty*

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Abstract. We look at the relationship between Adam's theory of conditionals and argumentation formalisms, to provide a principled characterisation of sound probabilistic semantics. This is a fundamental step towards having argumentation-based heuristics for probabilistic reasoning over uncertain probabilities encompassing aleatory and epistemic uncertainty.

Keywords: argumentation · aleatory uncertainty · epistemic uncertainty.

1 Problem Statement and Motivation

Probability is more fundamental than logic [5] because logic is a particular case within probability theory, where all probabilities are either 0 or 1; the converse is not valid. This suggests that formal approaches to rational argumentation should find some basis on probabilistic reasoning. We can see a probability assertion as a single-valued function that associates a numerical measure of belief—the probability—with a proposition—i.e. any well-formed formula in a formal theory. Formally,

$$P(F(x) \mid G(x), c) = z \qquad 0 \leq z \leq 1 \tag{1}$$

where $F(x)$ is the subject proposition, $G(x)$ is another proposition—typically a conjunction of propositions—used to "condition" the subject, and c is the general context. Such assertion is universally quantified, i.e. it can be equivalently be written as:

$$\forall(x)(\ P(F(x) \mid G(x), c) = z). \tag{2}$$

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Modifying the conditioning element $G(x)$, one can reach different conclusions about the subject proposition $F(x)$, thus capturing the idea of nonmonotonicity in plausible reasoning, where new pieces of evidence condition the evaluation of a given proposition.

This reasoning pattern naturally accommodates solutions to apparent paradoxes, such as the Monty Hall problem: here let us recall it briefly from [14], while an interested reader is referred to [13] for a complete—and amusing—discussion.

> Suppose you're on a game show, and you're given the choice of three doors. Behind one door is a car, behind the others, goats. You pick a door, say number 1, and the host, who knows what's behind the doors, opens another door, say number 3, which has a goat. He says to you, "Do you want to pick door number 2?" Is it to your advantage to switch your choice of doors?

Contrary to our intuition, the answer is a resounding yes, and it is very much dependent on the fact that the host opens a losing door.

Rational argumentation is based upon commonsense reasoning, which is richer than Bayesian probability and (nonmonotonic) logic [5], and it requires both. Bayesian learning and reasoning look more promising when dealing with inductive problems, nonmonotonic logic when dealing with (approximate) deductions. That is the challenge we believe is of relevance here: can formal argumentation approximate Bayesian inferences?

Such a question lies at the foundation for trustworthy artificial intelligence (AI). Successful human-AI teaming hinges on the human correctly deciding when to follow the recommendations of the AI system and when to override them [2]. We, therefore, need to empower AI agents to reason with and about (at least) two different sources of uncertainty: *aleatory*, and *epistemic* uncertainty [8]. Aleatory uncertainty refers to the variability in the outcome of an experiment, which is due to inherently random effects (e.g., flipping a fair coin): no additional source of information but Laplace's daemon can reduce such variability. Epistemic uncertainty refers to the agent's epistemic state using the model, hence its lack of knowledge that—in principle—can be reduced based on additional data samples. This is the case where Bayesian belief updates play the leading role. It can provide an estimation of the system's confidence—based on the accumulated amount of evidence for a given proposition—on the subject proposition.

Starting from the working hypothesis that formal argumentation can be of added value for human-AI teaming, it appears to us that it should be able to faithfully and rationally encompass Bayesian reasoning. Our intuition—which might prove to be wrong—is that formal argumentation can provide heuristics in dealing with certain types of plausible reasoning that concord with the results of Bayesian inferences beyond the case limits where all the values are either 0 or 1: we, however, have no answers yet to the limits and the costs of such heuristics.

2 Rationality and Probabilities

Given the ultimate goal of encompassing Bayesian reasoning in formal argumentation, it seems reasonable to begin by relying upon axioms of probabilistic inference in a formal theory. Following [9, p. 485], which bases upon [1], there are three fundamental rules of reasonable probabilistic inferences. Given ϕ, ψ, and η truth-functional formulas:

1. $(\phi \wedge \psi) \rightarrow \eta$ is a probabilistic consequence of $\phi \rightarrow \psi$ and $\phi \rightarrow \eta$;
2. $\phi \rightarrow \eta$ is a probabilistic consequence of $\phi \rightarrow \psi$ and $(\phi \wedge \psi) \rightarrow \eta$;
3. $(\phi \vee \psi) \rightarrow \eta$ is a probabilistic consequence of $\phi \rightarrow \eta$ and $\psi \rightarrow \eta$.

Neither transitivity nor contraposition is a theorem of such an axiomatic theory [9, p. 485], [1], while commonly adopted rationality postulates in argumentation [3] rely on them albeit only when dealing with strict rules only.

It is unclear whether such rules suffice for dealing with epistemic and aleatory uncertainty. For instance, when considering beta distributions as a formalism for representing both aleatory uncertainty (expected value) and epistemic uncertainty (variance) [4], it is unclear how to translate the semantics of ϵ-calculus that is at the basis of Adam's and Pearl's proposals [1, 9].

Finally, to fully support human-AI teaming, not only the AI system needs to be rational in its reasoning about uncertainty, but it also needs to express reasoning *about* uncertainty. For instance, we need to express statements such as *proposition X is true with degree T for which we have confidence C, because of our Bayesian analysis over data D that has been built on assumptions* $A_1, \ldots, A_n$., and such a statement should also have associated a probabilistic assessment of the conditional expressed by the *because*. Such statements will need to consider causal operators like Pearl's *do* [10] and will require adjustments to the argumentation semantics notions in the spirit of Prakken and Sartor's System II [12].

3 Conclusions and Related Problem Statements

In these preliminary considerations, we ask whether formal argumentation can provide heuristic reasoning tools aligned with Bayesian reasoning, particularly for encompassing both epistemic and aleatory uncertainty.

It is worth mentioning that there have been already approaches that tried to merge Bayesian reasoning and argumentation, albeit not with our goal. Indeed, Bayesian argumentation [6] emerged as a reaction to the MAXMIN rule for argumentation when combining linked and convergent arguments. When two or more independent arguments all support the same claim, we are in the presence of *convergent arguments*. Linked arguments instead form a chain of dependencies, thus providing support for a claim only in combination. For convergent arguments, Walton [15] argues in favour of the MAX rule, i.e. the overall strength or plausibility of the argument is determined by the maximum of the independent arguments converging to the same claim. For linked arguments, researchers

[15, 11] propose that its weakest link determines the overall plausibility of the argument. While some researchers [15] concede that there are cases where plausibility and probability are closely linked, others [7] contend that this is true in several instances. A probabilistic interpretation of the plausibility or strength of an argument leads to the conclusion that the MIN rule provides only an upper bound of the probabilistic interpretation of the strength of a linked argument.

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