On the use of conditional probability to model strength in logical argumentation

Pilar Dellunde[1,2], Lluís Godo[2], and Amanda Vidal[2]

[1] Universitat Autònoma de Barcelona and Barcelona Graduate School of Mathematics
Pilar.Dellunde@uab.cat
[2] Artificial Intelligence Research Institute (IIIA-CSIC),
Campus de la UAB, E-08193 Bellaterra, Barcelona, Spain
{godo,amanda}@iiia.csic.es

The objective of this contribution is to present work in progress in probabilistic logical argumentation. In [4] and [5] we introduced a framework inspired on the DeLP formalism, exploiting the notion of conditional probability. In that framework, we defined probabilistic arguments built from possibly inconsistent probabilistic knowledge bases, and the corresponding notions of attack, defeat and preference between these arguments. Then, we discussed consistency properties of admissible extensions of the Dung's abstract argumentation graphs obtained from sets of probabilistic arguments and the attack relations between them, showing that, in general, only one of the rationality postulates of [2] was satisfied.

The use of probability in argumentation can be traced back to Pollock's earliest works [12, 13]. Motivated by the formalization of epistemic reasoning, Pollock introduces the notion of strength of an argument in terms of numerical degrees of belief. Later, in [14], he addresses the question of how the degree of justification of a belief is determined. A conclusion may be supported by several different arguments, the arguments typically being defeasible, and there may also be arguments of varying strengths for defeaters for some of the supporting arguments. Some approaches to argumentation deal with the combination of argumentation-based inference and probability theory, one main open problem being how the strength of arguments is related to probability theory.

According to Prakken [15], two main approaches can be distinguished, depending on whether the uncertainty is *within* or *about* the arguments. When the uncertainty is *within* the arguments, probabilities are intrinsic to an argument in that they are used for capturing the strength of the argument given the uncertainty concerning the truth of its premises or the reliability of its inferences (see for instance [18]). When the uncertainty is *about* the arguments, probabilities are extrinsic to an argument in that they are used for expressing uncertainty about whether arguments are accepted as existing by some arguing agent (see for instance Hunter's constellation approach [8, 9]). In [15] the author also states that the intrinsic, or epistemic, approach can be applied in two ways: by simply computing probability values of conclusions or by using such probabilities to resolve attacks into defeats. In the recent work [18], arguments are generated in ASPIC+ and their rebutting attacks are resolved with probabilistic strengths of arguments. But some difficulties are encountered when assigning probabilities to arguments in an abstract framework. In a natural way, probabilities can be assigned to the truth of statements or to outcomes of events, but an argument is neither a statement nor an event. Hence, there is a need for a precise definition of what the probability of an argument is, and to do so, we have to work at the level of structured argumentation. For a first attempt to do so in the context of classical-logic argumentation see [8].

In a different direction, in [3] the authors propose a general scheme for adding probabilistic reasoning capabilities to a wide variety of knowledge representation formalisms. They add probabilities to the formulas of a given base logic, and define a probability distribution over the subsets of a knowledge base by taking the probabilities of the formulas into account. This gives rise to a probabilistic entailment relation that can be used for uncertain reasoning, see also [10]. This approach generalizes the ideas in ProbLog [6] and the constellation approach to the abstract argumentation in [8, 9].

The ongoing work treats the intrinsic case, assigning probabilities to arguments. In contrast to [8], but similarly to [16], we consider logic-based arguments $A = (support; conclusion)$, where *support* and *conclusion* are logical propositions, pervaded with uncertainty due a non-conclusive conditional link between their supports and their conclusions. In such a case, it is very reasonable to supplement the argument representation with a quantification α of how certain *conclusion* can be claimed to hold whenever *support* is known to hold [12], leading to represent an arguments as triples $A = (support; conclusion : \alpha)$. A very natural choice is to interpret α as some parameter related to the conditional probability $P(conclusion \mid support)$. This is, in a sense, in accordance with Bayesian epistemology [17], where they use conditional probabilities to quantitatively the support or confirmation of a hypothesis given an evidence. Here we consider α to be a probability interval $[c_1, c_2]$, meaning that the argument A provides the information $P(conclusion \mid support) \in [c_1, c_2]$.

If we internalise the conditional link within the argument as a conditional expression $support \rightsquigarrow conclusion$ and arguments get more complex and need several uncertain conditionals to link the support with the conclusion, then we can attach conditional probability intervals to each of the involved conditionals, so arguments become of the form

$$A = (\Pi, \Delta;\ \varphi \colon \alpha),$$

where $\Pi = \{\chi_1, \ldots, \chi_m\}$ is a finite set of factual (i.e. non conditional) premises χ_j's, $\Delta = \{(\psi_1 \rightsquigarrow \varphi_1 : \beta_1), \ldots, (\psi_n \rightsquigarrow \varphi_n : \beta_n)\}$ is a finite set of probabilistisc-valued conditionals, usually representing defeasible generic knowledge, and β_i's and α are probability intervals. In this settting, φ and all the $\chi_j, \psi_i, \varphi_i$'s are allowed to be arbitrary logical propositions. The interval α stands for the *argument strength*, and it is derived from the knowledge in the body of the argument as the minimum and maximum probabilities with which φ is entailed (for a given consequence relation) from Π and Δ. In fact, our definition of argument for a formula provides for both a logical argument and an optimal probabilistic entailment of its conclusion from its premises. In this aspect we follow [8] on decoupling the logic and the probabilistic aspects. More in detail, $A = (\Pi, \Delta;\ \varphi \colon \alpha)$ is an argument for φ with strength α when the following conditions hold:

- Probabilistic consistency: Δ is probabilistically satifiable, that is, there exists at least a probability P on formulas such that $P(\varphi_i \mid \psi_i) \in \beta_i$ for each $i = 1, \ldots, m$.
- Logical adequacy: φ logically follows (according to a suitable notion of logical consequence) from Π and the logical rules of Δ, i.e. $\{\psi_i \rightsquigarrow \varphi_i \mid i = 1, \ldots m\}$.

– c_1 (resp. c_2) is the infimum (resp. the supremum) of the values $P(\varphi \mid \Pi^\wedge)$ when letting P vary among the probabilities satisfying Δ.
  In other words,

  $c_1 = \sup\{d_1 \in [0,1] \mid \Delta \models_{Pr} \Pi^\wedge \rightsquigarrow \varphi\colon [d_1, 1]\}$,
  $c_2 = \inf\{d_2 \in [0,1] \mid \Delta \models_{Pr} \Pi^\wedge \rightsquigarrow \varphi\colon [0, d_2]\}$,

  where $\Pi^\wedge$ denotes the conjunction of the propositions in Π, and $\models_{Pr}$ denotes the following notion of probabilistic entailment [10]:

$$\{(\psi_1 \rightsquigarrow \varphi_1 : \beta_1), \ldots, (\psi_n \rightsquigarrow \varphi_n : \beta_n)\} \models_{Pr} (\Pi^\wedge \rightsquigarrow \varphi\colon \alpha) \quad \text{if}$$
$$\text{for any probability } P, \text{ if } P(\varphi_i \mid \psi_i) \in \beta_i \text{ for all } i = 1, \ldots, n, \text{ then } P(\varphi \mid \Pi^\wedge) \in \alpha.$$

– Δ is minimal satisfying the above conditions.

This type of arguments can be seen as a probabilistic generalization of those at work in the Defeasible Logic Programming argumentation framework (DeLP) [7].

Using a generalization of the usual notions of subargument and disagreement we can introduce the notion of attack. Finally, we can specify when an attack is actually a defeat. The probability intervals attached to the arguments is a determinant factor for this. However, relying only on the involved weights to decide when an argument prevails over another can be counter-intuitive in some situations. For instance, consider the following example of two conflicting arguments. The following arguments encode the scenario of inferring whether a damage (a big monetary loss) is caused in a house when it suffers a burglary (with probability in [0.9, 1]). However in the case of a burglary, the monetary loss can be avoided if there is an alarm in the house and the alarm is triggered (with probability in [0.8, 1]):

$$A_1 = (\{burglary\}, \{burglary \rightsquigarrow mon\_loss\colon [0.9, 1]\}; mon\_loss\colon [0.9, 1])$$
$$A_2 = (\{burglary, alarm\}, \{burglary \wedge alarm \rightsquigarrow \neg mon\_loss\colon [0.8, 1]\}; \neg mon\_loss\colon [0.8, 1])$$

There, the derived probability of an argument A_2 is smaller than the one derived in a second argument A_1, but the argument A_2 was based on more information than A_1. Thus, a specificity criterion seemed more adequate in these cases. Nevertheless, in case of two conflicting arguments using the same amount or incomparable information, the probability attached to the arguments becomes a suitable criterion to use. In order to compare two arguments, we will then give the specificity criterion the highest priority, and if it does not produce a proper comparison, the degrees of probability will determine the strongest argument. This decision is based on the fact that probabilities of the defeasible rules inside each argument, and of the argument itself, reflect the faithfulness of each rule and indirectly, of the argument itself.

An important question we plan to deal with in future work concerns which is the most adequate way of modelling the evidence in this setting. Namely, this involves determining whether is better to model evidential knowledge by assuming that the (probabilistic) reasoning is conditioned by the evidence, as it is assumed above, or by assigning to those factual and conditional pieces of knowledge probability equal to 1. Also, in future work we aim at exploring in deeper detail the trade-off between specificity and probability criteria. Moreover, taking into account that only one of the rationality postulates of [2] is satisfied, the consideration of a notion of collective conflict, similarly to the one in [1] seems promising. Finally, we would like to study further whether the probabilities involved in the arguments could allow for gradual notions of attack and acceptability.

Acknowledgments The authors thank the reviewers for their useful comments, and acknowledge partial support by the European Project H2020-101007627-MSCA-RISE-MOSAIC, and the Spanish project PID2019-111544GB-C21.

References

1. T. Alsinet, R. Béjar, L. Godo, F. Guitart. RP-DeLP: a weighted defeasible argumentation framework based on a recursive semantics. *J. Log. Comput.* 26(4): 1315-1360, 2016.
2. M. Caminada and L. Amgoud. On the evaluation of argumentation formalisms. *Artificial Intelligence*, 171:286–310, 2007.
3. F. Cerutti and M. Thimm. A general approach to reasoning with probabilities. *International Journal of Approximate Reasoning*, 111:35–50, 2019.
4. P. Dellunde, L. Godo, and A. Vidal. Probabilistic Argumentation: An Approach Based on Conditional Probability -A Preliminary Report. In Proc. JELIA'21, *LNAI*, vol 12678, 25-32, 2021.
5. P. Dellunde, L. Godo, and A. Vidal. On probabilistic logical argumentation based on conditional probability. In Proc. CCIA'21, in press.
6. D. Fierens and G. van den Broeck and J. Renkens and D. Shterionov and B. Gutmann and I. Thon and G. Janssens and L. De Raedt Inference and learning in probabilistic logic programs using weighted Boolean formulas. *Theory and Practice of Logic Programming*, 15(3):358–401, 2015.
7. A. Garcia and G. Simari. Defeasible logic programming: an argumentative approach. *Theory and Practice of Logic Programming*, 4(1-2):95–138, 01 2004.
8. A. Hunter. A probabilistic approach to modelling uncertain logical arguments. *International Journal of Approximate Reasoning*, 54(1):47–81, 2013.
9. A. Hunter. Probabilistic qualification of attack in abstract argumentation. *IJAR*, 55(2):607–638, 2014.
10. T. Lukasiewicz. Probabilistic logic programming with conditional constraints. ACM Transactions on Computational Logic, 2(3):289-339, 2001.
11. S.H. Nielsen and S. Parsons An application of formal argumentation: using Bayesian networks in multi-agent systems. Artificial Intelligence, 171:754–775, 2007.
12. J.L. Pollock. Justification and defeat. *Artificial Intelligence*, 67:377–408, 1994.
13. J.L. Pollock. *Cognitive carpentry. A blueprint for how to build a person*. MIT Press, Cambridge, MA, 1995.
14. J.L. Pollock. Defeasible reasoning with variable degrees of justification. *Artificial Intelligence*, 133:233–282, 2002.
15. H. Prakken. Historical overview of formal argumentation. In P. Baroni, D. Gabbay, M. Giacomin, and L. van der Torre, editors, *Handbook of Formal Argumentation*, volume 1, pages 73–141. College Publications, 2018.
16. H. Prakken. Probabilistic strength of arguments with structure. In M. Thielscher et al. (eds.), Proc. of KR 2018, pages 158–167. AAAI Press, 2018.
17. W. Talbott. Bayesian Epistemology. In Edward N. Zalta, editor, *The Stanford Encyclopedia of Philosophy*. Metaphysics Research Lab, Stanford University, winter 2016 edition, 2016.
18. S. Timmer, J.J.Ch. Meyer, H. Prakken, S. Renooij, and B. Verheij. A two-phase method for extracting explanatory arguments from bayesian networks. *Int. J. Approx. Reason.*, 80:475–494, 2017.